6.897 Advanced Data Structures (Spring'05)

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Problem 9 – Solution

1. If we find two consecutive points of the same color, we can eliminate the second one, and the result of any predecessor query will not change. Thus, we can assume colors alternate in the sorted list of points. By symmetry, assume the first point is red. For every i, create a segment between the $2i$-th and $(2i+1)$-st points (observe that the segments are disjoint). Now, if x stabs a segment, it's predecessor has an even index, so it is blue. Otherwise, the predecessor is red.

2. Replace a segment $[a, b]$ by the point $(a, u-b)$ in two dimensions. Now, for some x, we query the range $[0, x] \times [0, u-x]$. We have $(a, u-b) \in [0, x] \times [0, u-x] \iff a \leq x, u-b \leq u-x \iff a \leq x \leq b \iff x \in [a, b]$.

3. In the preprocessing phase, compute the Euler tour traversal of the tree. For each node, remember the index of the first and last appearance of the node. Leaves appear only once. To mark a node, we insert a segment whose endpoints are the indices of the first and last appearance in the Euler tour. To unmark a node, we delete that segment. To query a leaf v, do a segment stabbing query at the (unique) point where v appears in the Euler tour. A marked ancestor is equivalent to a segment beginning before v's appearance, and ending after it (by definition of the Euler tour). Thus, v's associated point stabs the segment associated to any marked ancestor.