6.897 Advanced Data Structures (Spring'05)

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Problem 3 – Solution

We hold two data structures D_1 and D_2. For the first $\frac{m}{2}$ operations, we simulate all operations on both D_1 and D_2, taking time $2t$ per operation. After this, the data structures will have a phase shift of $\frac{m}{2}$ operations.

During the next $\frac{m}{2}$ operations, we use D_1 as the main data structure. We run operations on it, and obtain the relevant results. During this time, we perform a global rebuilding on D_2, followed by a simulation of all the $\frac{m}{2}$ operations which it missed (but we stored them somewhere for later use). This takes time $mt + \frac{m}{2}t = \frac{3}{2}mt$. We simulate $3t$ steps of this process for every operation run on D_1. Thus, the worst-case running time is $4t$ per operation.

At the end of this, D_2 has caught up with all past operations, and it can handle new operations as the main data structure. At this time, D_1 must go into global rebuilding, so it becomes the secondary data structure. After $\frac{m}{2}$ operations, we switch roles again. In the steady state, a data structure has a global rebuilding after exactly m operations: the first $\frac{m}{2}$ are not in real time (they are executed while it is catching up), and the next $\frac{m}{2}$ use it as the main data structure.