Predecessor problem:

6.897
L9.2
Mar. 3, 2005

maintain a set S of n elements $\in \mathcal{U}$
(w-bit integers $\in \mathcal{U}$)

subject to
- insert(x), $x \in \mathcal{U} - S$
- delete(x), $x \in S$
- predecessor(x), $x \in \mathcal{U}$
  - ↳ previous element in S before x
- successor(x), $x \in \mathcal{U}$
  - ↳ symmetric

"Classic" results: (we'll see the "right answer" later)

(recent)

		Time/op	Space	Model
	Balanced trees (1962)	$O(\lg n)$	$O(n)$	BST + comparison
		$\Omega(\lg n)$	*	BST or comparison
TODAY	van Emde Boas (1975)	$O(\lg w) = O(\lg \lg u)$	$O(u)$	transdich. RAM (word & AC^0) [& pointer machine]
TODAY	y-fast trees (Willard 1984)	$O(\lg w)$ w.h.p.	$O(n)$	transdich. RAM (word)
NEXT LECTURE	fusion trees (Fredman & Willard 1993)	$O(\lg_w n) = O\left(\frac{\lg n}{\lg \lg u}\right)$	$O(n)$	transdich. RAM (word & AC^0) ↓ [Andersson, Miltersen, Thorup – TCS 1999]

Combination: $O(\sqrt{\lg n})$

- if $\lg w \le \sqrt{\lg n}$ then $O(\lg w) = O(\sqrt{\lg n})$
  - ↳ van Emde Boas or y-fast trees
- if $\lg w \ge \sqrt{\lg n}$ then $O(\lg_w n) = O\left(\frac{\lg n}{\lg w}\right) = O(\sqrt{\lg n})$
  - ↳ fusion trees