Dynamic directed graphs & shortest paths

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Transitive closure: is there a path from v to w?

- bulk update: insert/delete vertex & incident edges

- bulk update: $O(n^2)$ am. [Demetrescu & Italiano - FOCS 2000]
  query: $O(1)$ w.c. & [Roditty - SODA 2003]

- same bounds, worst case [Sankowski - FOCS 2004] ⇒ TODAY

- optimal if storing transitive closure matrix explicitly

OPEN: $o(n^2)$ update worst case?

- bulk update: $O(m\sqrt{n} \cdot t)$ am. [Roditty & Zwick - FOCS 2002]
  query: $O(\sqrt{n}/t)$ w.c., $t = O(\sqrt{n})$

- bulk update: $O(m + n \lg n)$ am. [Roditty & Zwick - STOC 2004]
  query: $O(n)$ w.c.

OPEN: full trade-off with update × query $= O(mn)$ or $O(n^2)$

- acyclic: update: $O(n^{1.575} \cdot t)$ [Demetrescu & Italiano - FOCS 2000]
  query: $O(n^{0.575}/t)$, $t = O(n^{0.325})$

- decremental: update: $O(n)$ am. [ditto]
  query: $O(1)$ w.c.

All-pairs shortest paths: weight of shortest path from v to w?

- bulk update: $O(n^2(\lg n + \lg^2(1 + m/n)))$ am. [Thorup - SWAT 2004]
  query: $O(1)$ w.c. (improving on [Demetrescu & Italiano - STOC 2003])

OPEN: $O(n^2)$ or $o(n^2)$ update, even for undirected graphs!

- worst case: update: $O(n^{2.75})$ [Thorup - STOC 2005]
  query: $O(1)$

- unweighted: update: $O(m\sqrt{n}\ \mathrm{polylg}\ n)$ am. [Roditty & Zwick - ESA 2004]
  query: $O(n^{3/4})$ w.c.

- undirected, unweighted, and $(1+\varepsilon)$-approximate:
  update: $O(\sqrt{m}\, n \cdot t)$ am. [Roditty & Zwick - FOCS 2004]
  query: $O(\sqrt{m}/t)$ w.c., $t = O(\sqrt{n})$

[- incremental & decremental results skipped]