6.897
L7.5
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Final part of encoding:

"separator" for $R \setminus W$, $W \setminus R$

Separators:

- universe $\mathcal{U}$, number m
- separator family S for size-m sets if
  $\forall A, B \subseteq \mathcal{U}$ with $|A|, |B| \le m$ & $A \cap B = \emptyset$:
  $\exists C \in S$ such that $A \subseteq C$ & $B \subseteq \mathcal{U} \setminus C$

- claim: there is a separator family S for size-m sets such that $|S| \le 2^{O(m + \lg\lg \mathcal{U})}$

  proof: use perfect hashing / bloomier filter which takes $O(\lg m + \lg\lg u)$ bits & maps A & B to table of size $O(m)$ & store bit vector saying A vs. B for table. □

Final decoding:

- simulate all possible queries
- claim: incorrect query ⇒ NO or ABORT
  - cells read in right subtree by incorrect query written in right subtree → easy
    in left subtree → test if in $W \cap R$ → use that value
    test if in $W \setminus R$ i.e. wrong side of separator → ABORT
    in past → use known value

approx. but good enough

Size of encoding:

$|R \cap W| \cdot O(\lg n) + O(|R| + |W| + \lg\lg n)$

$\Rightarrow \Omega(\ell\sqrt{n} \lg n)$ by info. theory

$|R| + |W| = o(\ell\sqrt{n} \lg n) \Rightarrow$ contradiction □