Lower bounds

6.897
L 7.1
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Cell-probe model:

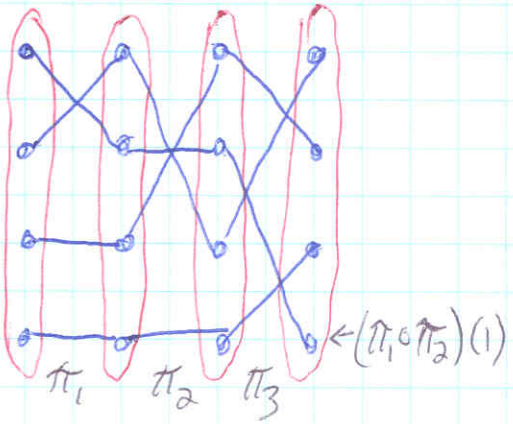
- w-bit cells (decomp. of memory)
- just count # reads & writes
- computation is free
- typically assume $w = \Theta(\lg n)$

Dynamic connectivity lower bound [Patrascu & Demaine - SODA2004 & STOC2004]

$\Omega(\lg n)$ in worst case [here]
amortized [in paper]
on cell-probe model

Proof:

- consider $\sqrt{n} \times \sqrt{n}$ grid with perfect matching between consecutive columns
  → permutation π_i



- block operations:
  - update (i, π): $\pi_i \leftarrow \pi$
    ↳ $O(\sqrt{n})$ edge insertions/deletions
  - verify-sum (i, π): $\sum_{j=1}^{i} \pi_j = \pi$? (Σ = compose)
    ↳ $O(\sqrt{n})$ connectivity queries

- **Claim:** $\sqrt{n}$ updates + $\sqrt{n}$ verify-sums require $\Omega(\sqrt{n} \cdot \sqrt{n} \cdot \lg n)$ time
  ⇒ dynamic connectivity requires $\Omega(\lg n)$ time.