Other dynamic graph problems than connectivity

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- minimum spanning forest (MST ∀ connected component) (represented, say, as dynamic tree)
  - $O(\lg^4 n)$ update [Holm, de Lichtenberg, Thorup – JACM 2001]
  - worst case: $O(\sqrt{n})$ update [Eppstein et al. – JACM 1997]
  - plane graphs: $O(\lg n)$ [Eppstein et al. – JCSS 1992]

- bipartiteness: is the graph bipartite? (2-colorable)
  - reducible to minimum spanning forest

- k-connectivity: (vertex or edge)
  - between pairs of vertices:
    - $O(\text{polylg } n)$ for $k=2$ [Holm et al. – JACM 2001]
    - planar & decremental: [Giammarresi & Italiano – Algorithmica 1996]
      $O(\lg^2 n)$ for 3-edge-conn.
    - worst case: [Eppstein et al. – JACM 1997]
      $O(\sqrt{n})$ for 2-edge-conn.
      $O(n)$ for 2-vertex-conn. & 3-vertex-conn.
      $O(n^{2/3})$ for 3-edge-conn.
      $O(n\alpha(n))$ for $k=4$
      $O(n \lg n)$ for $O(1)$-edge-conn.
    - OPEN: polylg for $k=O(1)$? $k=\text{polylg}$?
  - of whole graph: ($\sim$ min cut = max flow)
    - $O(\sqrt{n}\,\text{polylg } n)$ for $O(\text{polylg } n)$-edge-conn. & min cut up to that size [Thorup – STOC 2001]
    - OPEN: polylg for $k=O(1)$? $k=\text{polylg}$?

- planarity testing: does inserting edge e violate planarity? ($\Rightarrow$ can't insert)
  - $O(n^{2/3})$ [Galil, Italiano, Sarnak – JACM 1999]
  - $O(\lg^2 n)$ worst case for fixed embedding [Italiano et al. – ESA 1993]
  - $O(\alpha(m,n)\cdot m + n)$ total for m ops., incremental [La Poutre – STOC 1994]

OPEN: testing whether graph has a fixed minor?
($\Rightarrow$ any minor-closed graph property)