Dynamic (undirected) graph problems: (directed later)

6.897
L6.2
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- maintain graph subject to
  - insert edge (v, w) } primary
  - delete edge (v, w) }
  - [insert vertex
  - delete vertex v with no incident edges]
  - some query

- connectivity(v, w): is there a path from v to w?
  or (G): is the graph connected?
  - $O(\lg n)$ for trees [as we've seen]
  - $O(\lg n)$ for plane graphs [Eppstein et al. – JCSS 1992]
  - OPEN: $O(\lg n)$ for general graphs?
  - amortized:
    - $O(\lg n \cdot (\lg\lg n)^3)$ update [Thorup – STOC 2000]
      $O(\lg n / \lg\lg\lg n)$ query
    - $O(\lg^2 n)$ update [Holm, de Lichtenberg, Thorup – JACM 2001]
      $O(\lg n / \lg\lg n)$ query ⟹ TODAY
    - $O(\lg n \cdot x)$ update $\Rightarrow$ [Pătraşcu & Demaine – STOC 2004]
      $\Rightarrow \Omega(\lg n / \lg x)$ query, for $x \geq 1$ ⟹ Lecture 7
      [$O(\lg n \cdot x)$ query $\Rightarrow \Omega(\lg n / \lg x)$ update, for $x \geq 1$]
      - Thorup & Holm et al. are on this trade-off curve
      - OPEN: $o(\lg n)$ update & $O(\text{poly}\lg n)$ query?
      - OPEN: $O(\lg n)$ update & query?
  - worst case: OPEN: poly log?
    $O(\sqrt{n})$ update [Eppstein, Galil, Italiano, Nissenzweig – JACM 1997]
    $O(1)$ query
    - also on trade-off curve from lower bound
  - incremental: insertions, no deletions
    - amortized: $O(\alpha)$ via union-find
    - worst case: $\Theta(x)$ updates $\Rightarrow \Theta(\lg n / \lg x)$ queries (upper & lower bounds)
  - decremental: deletions, no insertions
    $O(m \lg n + n \,\text{polylog}\, n)$ for all updates [Thorup – JACM 1999]
    $O(1)$ query (m = # edges)