$O(\lg n)$ bound (amortized)

- $s(v)$ = # nodes in v's subtree in tree of auxiliary trees
  = # nodes in v's subtree within v's auxiliary tree
  + $\sum$ # nodes in each "descendant" auxiliary tree of these nodes
- potential $\Phi = \sum_v \lg s(v)$

Claim: amortized cost of access
$= O(\lg n) + O(1) \times$ # preferred child changes
$(= O(\lg n)$ amortized$)$

Proof:

usual splay analysis (Access theorem)

- splay(v) costs $\le 3(\lg s(u) - \lg s(v)) + 1$
  where u = root of v's auxiliary tree
- splay(v) affects s's only within v's aux. tree
- changing v's preferred child doesn't change s's because tree of aux. trees stays same
- $s(v) \le s(w)$
  ↳ next element to splay

$\Rightarrow$ telescopes

$\Rightarrow$ access cost $\le 3(\underbrace{\lg s(r) - \lg s(v)}_{O(\lg n)}) + O(\#\text{pref. child changes})$

- cut(v) only decreases s's $\Rightarrow \Phi$ only decreases
- join(v, w) just increases $s(v)$ ~ by at most n
  $\Rightarrow O(\lg n)$ increase in Φ □