$O(\lg^2 n)$ bound (amortized)

- link & cut cost $O(1)$ beyond access
- findroot costs access + time to find & splay min.
- access costs time to splay × # preferred child changes
- splay analysis works in this setting (splits/concatenates)

$\Rightarrow O(\lg n) \times$ total number of preferred child changes
↳ claim: $O(m \lg n)$

Heavy-light decomposition (in represented tree)

- size(v) = # nodes in v's subtree
- edge (v, parent(v)) is heavy if size(v) > ½ · size(parent(v))
  & light otherwise
- at most one heavy child of a node
- heavy paths partition the nodes of the tree
- light depth of node = # light edges on path to root
  $\leq \lg n$.

Bound on preferred child changes

- each change makes an edge preferred
- during access(v), these edges lie on path from v to root
  $\Rightarrow \leq \lg n$ of them are light
- how many heavy preferred edges created?
- count number of heavy preferred edges destroyed and add $\leq n-1$ for the end config.
- access destroys heavy pref. edge by making it unpreferred & making a light edge preferred (except 1 at bottom)
  $\Rightarrow \leq \lg n + 1$
- link(v, w) heavens nodes on path from w to root
  $\Rightarrow$ edges incident to this path may become light
  but access(w) made these edges unpreferred
  $\Rightarrow$ no destructions
- cut(v) lightens ancestors of v
  but $\leq \lg n$ of them can become light
  +1 destruction for the edge (v, parent(v)) deleted