Key-independent optimality [Iacono – ISAAC 2002]

- suppose key values & ranks are "meaningless"
  ⇒ might as well take uniformly random bijection b on keys
- $\mathop{E}_{b}[OPT(b(x_1), b(x_2), \ldots, b(x_m))] = \text{working-set bound} = \Theta\left(\sum_{i=1}^{m} \lg t_i(x_i)\right)$
  (⇒ splay trees, etc. are key-independently optimal)

Proof: (sketch)

- working set is always an upper bound ⇒ O
- claim: Wilber 2 is equal to working-set bound ⇒ Ω
  (expected)
  - wilber(x_j) is all about the working set of x_j
    i.e. distinct elements accessed since last access to x_j
    - x_i stops at last access to x_j
    - if an element is repeated, only the later access matters (discard earlier for x_j)
  - bijection gives this working set (W) a random permutation (because accesses now unique)
  - x_j falls roughly in middle
  - $E[\#\text{ times } a_i \text{ changes}] = \Theta(\lg |W|)$ ← like # times max changes
  - $E[\#\text{ times } b_i \text{ changes}] = \Theta(\lg |W|)$ ← min
  - constant fraction of these changes will interleave

⇒ wilber(x_j) $= \Theta(\lg |W|) = \Theta(\lg t_j(x_j))$.