Bounds

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L3.4
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Consequences of access theorem:

- **log amortized:** $w_i = 1 \ \forall i \Rightarrow s(\text{root}) = n$
  $\Rightarrow O(\lg \frac{n}{\geq 1}) = O(\lg n)$
- **static optimality:** $w_i = p_i = f_i/m \Rightarrow s(\text{root}) = 1$, $s(x) \geq w_x = p_x$
  $\Rightarrow O(\lg \frac{1}{\geq p_x}) = O(\lg 1/p_x)$
- **static finger theorem:** fix your "finger" on a node f.
  $w_i = \frac{1}{(1+i-f)^2} \Rightarrow s(\text{root}) \leq \sum_{k=-\infty}^{\infty} \frac{1}{k^2} = \pi^2/3$
  $\Rightarrow O(\lg \frac{O(1)}{w_i}) = O(\lg (1+i-f)^2) = O(\lg (1+|i-f|))$
  dist. to finger
- **working-set theorem:** $O(1+\lg t_i(x_i))$ amortized to access x_i
  where $t_i(y) =$ # distinct elements accessed since last access to y before time i, including y
  plus startup cost of $O(n \lg n)$ (or assume $m \geq n \lg n$)
  - w_y at time $i = 1/t_i(y)^2 \Rightarrow w(\text{root}) \leq \sum_{k=0}^{\infty} \frac{1}{k^2} = \pi^2/6$
  - upon access to x_i, pay $O(1+\lg t_i(x_i)^2) = O(1+\lg t_i(x_i))$,
    change $t_i(x_i)$ to 1 — (increasing w_{x_i} to 1)
    increment $t_i(y) <$ old $t_i(x_i)$ — (decreasing w_y slightly)
  - sizes only decrease: only w_{x_i} increases but it becomes root & size(root) is fixed

  $\Rightarrow$ potential only decreases
  - max. potential decrease $= O(n \lg n)$.