Splay trees [Sleator & Tarjan – JACM 1985]

6.897
L3.3
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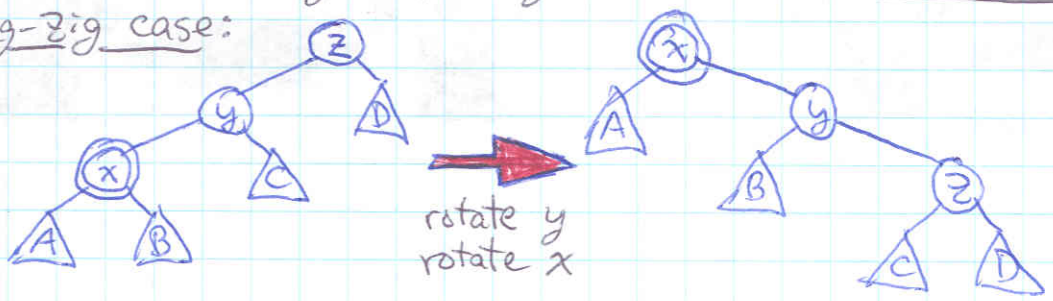
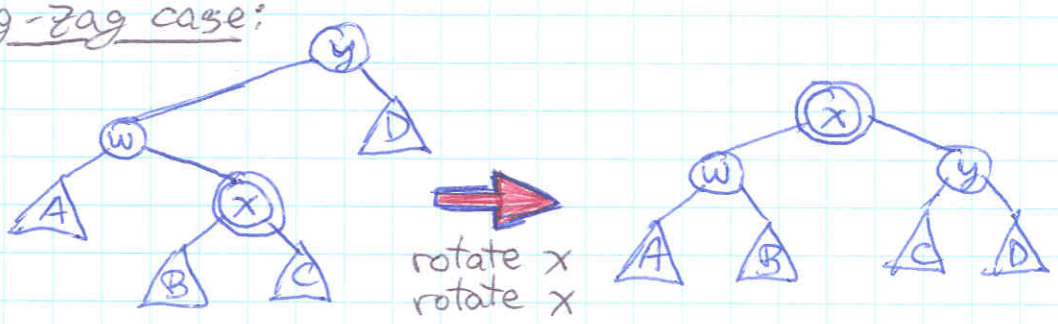
- walk to query element x
- move it to root by repeating:
  - zig-zig case:



  - zig-zag case:



  - at end, may have to do a single rotation

CONJECTURE: dynamically optimal
- known: statically optimal

Access theorem: just count rotations

- assign arbitrary weight w_i to each element i
- define $s(x)$ for node x to be $\sum \{ w_i \mid i \text{ in subtree rooted at } x \}$
- potential function $= \sum_x \lg s(x)$ → low ⇒ balanced; → high ⇒ unbalanced
- amortized cost of one splay step (zig-zig or zig-zag)
  $\leq 3 (\lg s^{new}(x) - \lg s^{old}(x))$
  (some checking + concavity of lg)
- amortized cost of accessing x telescopes:
  $\leq 3 (\lg s^{new}(x) - \lg s^{old}(x)) + 1$
  for final rotation
- max. potential change $\leq \sum$ these costs for all nodes
  [min $\Phi = \sum_i \lg w_i$; max $\Phi = n \lg \sum_i w_i$; diff $\leq \sum_i (\lg \sum_j w_j - \lg w_i)$] in initial tree