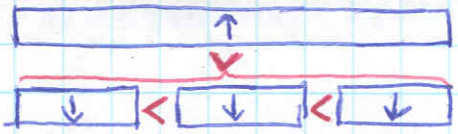
6.897
L24.5
May 5, 2005

Invariants:



- keys in down buffers < up buffer @ same level
- keys in down buffers @ $X^{3/2}$ < down buffers @ $X^{9/4}$ above
- order between down buffers: keys in first < second < ...

Analysis:

- claim: push/pull at level $X^{3/2}$ (only) $= O(\frac{X}{B} \lg_{M/B} \frac{X}{B})$ memory transfers
- assume all levels of size $\leq M$ stay in cache $\Rightarrow$ free
- tall-cache assumption: $M \geq B^2$.
- push at level $X^{3/2} > B^2 \Rightarrow X > B^{4/3}$ ($\Leftrightarrow X/B > 1$)
  - sort costs $O(\frac{X}{B} \lg_{M/B} \frac{X}{B})$ memory transfers
  - distribute costs $O(X/B + \sqrt{X})$ memory transfers
    (scan; startup per down buffer)
  - if $X \geq B^2$ then cost $= O(X/B)$ ✓
  - else: only one such level with $B^{4/3} < X \leq B^2$
    can keep 1 block per down buffer ($\sqrt{X} \leq B \leq \frac{M}{B}$; tall cache)
    so just pay $O(X/B)$
- pull at level $X^{3/2} > B^2$:
  - sort costs $O(\frac{X}{B} \lg_{M/B} \frac{X}{B})$ memory transfers
  - another sort of $\sqrt{X^{3/2}}$ elts.
    only when recursively pulling — charge to that. □

- X elts. involved in push/pull costing $O(\frac{X}{B} \lg_{M/B} \frac{X}{B})$
- each elt. goes up & then down [more or less] (real proof messier)

$\Rightarrow O(\frac{1}{B} \sum_{X} \lg_{M/B} \frac{X}{B})$ amortized cost per element

(geometric; exp. geometric)

$= O(\frac{1}{B} \lg_{M/B} N/B)$