Cache-oblivious priority queues

[Arge, Bender, Demaine, Holland-Minkley, Munro - STOC 2002]

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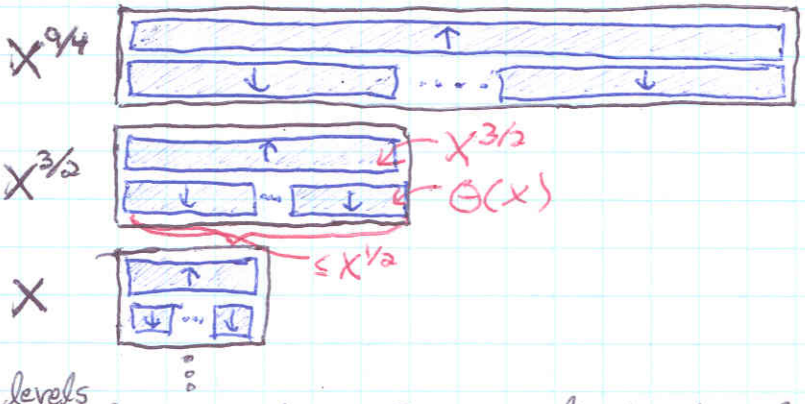
- $\lg \lg n$ levels of size $N, N^{2/3}, N^{4/9}, \ldots, c$
- level $X^{3/2}$ has 1 up buffer of size $X^{3/2}$ & $\leq X^{1/2}$ down buffers each of size $\Theta(X)$
  (except first, each contains $\Theta(X)$ elts.)



levels

- stored in order from smallest to largest (say)
- invariants on next page

Insert:
- append to smallest up buffer.
- swap with largest elt. in smallest down buffer if appropriate
- if up buffer overflows: push

Push X elts. into level $X^{3/2}$ s.t. keys > all elts. in down buffers of level X below (up buffer, when we reach large elts.)
- sort elts.
- distribute among down buffers (scanning elts. & visiting buffers in order)
- when down buffer overflows: split in half, link together
- when # down buffers overflows: move last down buffer up to up buffer
- when up buffer overflows, push it up to $X^{9/4}$

Delete-min:
- if smallest down buffer underflows: pull
- return smallest elt. in smallest down buffer

Pull X smallest elts. from level $X^{3/2}$ and above
- if $\geq X$ elts. in down buffers, sort first two down buffers & extract
- else:
  - pull $X^{3/2}$ smallest elts. from $X^{9/4}$ above
  - sort these elts. & up buffer
  - put larger elts. in up buffer (same # as before)
  - take what's needed beyond old down buffers to get X smallest elts.
  - distribute rest into down buffers