6.897
L24.3
May 5, 2005

List labeling:

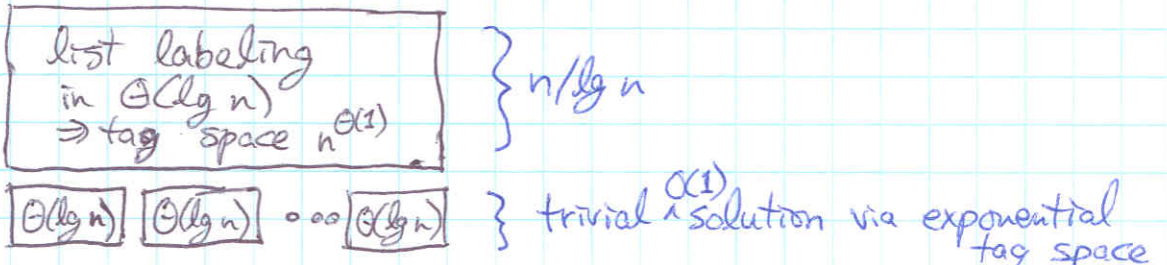
- goal: maintain explicit tag for each element such that tags are monotone through list subject to "linked list updates" — insert & delete here

tag space	time per update	
$(1+\varepsilon)n$ ... $\Theta(n \lg n)$	$O(\lg^2 n)$	≡ ordered file maintenance
$n^{1+\varepsilon}$ ... $n^{O(1)}$	$\Theta(\lg n)$	~O via modified thresholds ~Ω [Dietz, Seiferas, Zhang – SODA 2005]
2^n	$\Theta(1)$	~trivial

Order queries in list: easier

- goal: maintain linked list subject to updates (insert/delet here) & query: is node x before node y?

- $O(1)$ solution via indirection: [Dietz & Sleator – STOC 1987] [Bender, Cole, Demaine, Farach-Colton, Zito – ESA 2002]



- implicit tag of element = (tag from top, tag from bottom)
  - tag from top: $n^{\Theta(1)}$ options
  - tag from bottom: $n^{\Theta(1)}$ options

⇒ polynomial tag space (but tags are implicit)
⇒ can compare two elements in constant time
- bottom layer slows updates to top by factor of $\Theta(\lg n)$
⇒ $O(1)$ amortized update

- worst-case bounds also possible [same refs.]