Cache-oblivious binary search / static search trees.

Van Emde Boas layout [Prokop-MEng 1999] [Bender, Demaine, Farach-Colton-FOCS 2000]

6.897
L23.3
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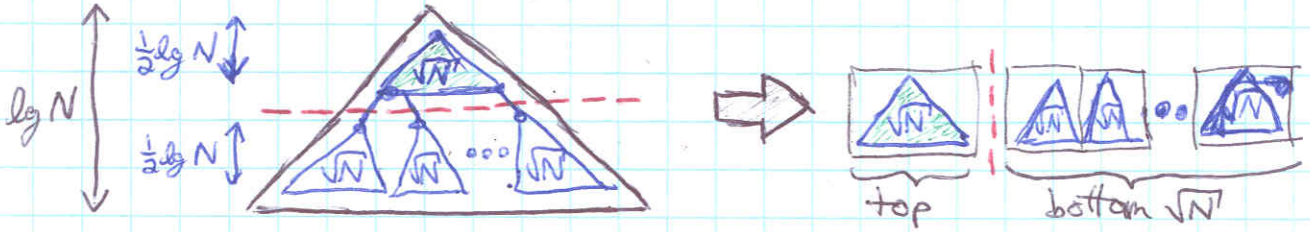
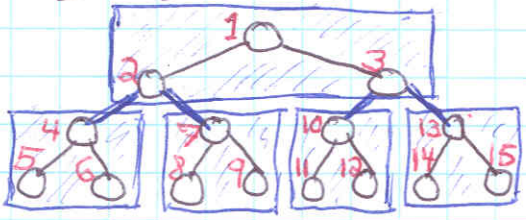
- store N elements in order in N-node complete binary tree
- carve tree at middle level of edges
  $\Rightarrow \sqrt{N}+1$ pieces of $\sqrt{N}$ each



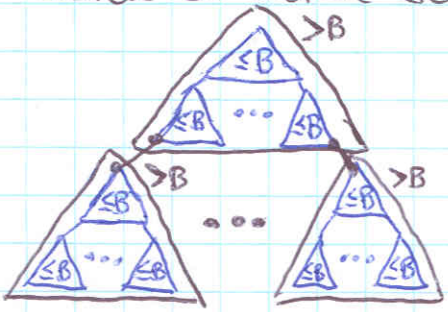
- recursively lay out pieces & concatenate

e.g.



Analysis:

- consider level of detail (refinement) straddling B:



- cutting height in half until $\leq \lg B$
  $\Rightarrow$ pieces have height between $\frac{1}{2}\lg B$ & $\lg B$
  (& nodes between $\sqrt{B}$ & B)
- # pieces visited on root-to-leaf path $\leq \frac{\lg N}{\frac{1}{2}\lg B} = 2\lg_B N$
  $\leftrightarrow$ at this level of detail
- each piece has $\leq B$ elements & is stored consecutively
  $\Rightarrow$ occupies ≤ 2 blocks (depending on alignment)
  $\Rightarrow$ # memory transfers in search $\leq 4\lg_B N$

+1 no pieces

Generalizes to
- height $\neq$ power of 2
- node degrees $\neq 2$ but still ≥ 2 & $O(1)$