Succinct data structures (usually static)

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- implicit DS: space = information-theoretic optimum + $O(1)$ bits ← for rounding
  - typically, DS is "just the data", in some order
- succinct DS: space = information-theoretic optimum + o(that)
- compact DS: space = O(information-theoretic optimum)

Survey:

- implicit dynamic search tree [Franceschini & Grossi — ICALP 2003 & WADS 2003]
  $O(\lg n)$ w.c. per Insert/Delete/Search (cf. heap, sorted array)
  comparison model on elts.; word RAM for ptrs.
- succinct dictionary [Brodnik & Munro - SICOMP 1999; Pagh - SICOMP 2001]
  $\lg \binom{u}{n} + O\left(n \frac{(\lg \lg u)^2}{\lg u}\right)$ bits, $O(1)$ membership query
- succinct binary trie [Munro & Raman - SICOMP 2001]
  $C_n = \binom{2n}{n}/(n+1) \sim 4^n$ such tries
  $\lg C_n + o(\lg C_n) = 2n + o(n)$ bits
  $O(1)$ left child, right child, parent, subtree size
- compact/succinct k-ary trie [Benoit, Demaine, Munro, Raman, Raman, Rao — Algorithmica 2005]
  $C_n^k = \binom{kn+1}{n}/(kn+1)$ such tries
  $\lg C_n^k \sim (\lg k + \lg e)n$
  $(\lceil \lg k \rceil + \lceil \lg e \rceil)n + o(n)$ bits
  $O(1)$ child with label i, parent, subtree size
- succinct rooted ordered tree [Benoit et al. 2005]
  only C_n such trees
  $2n + o(n)$ bits
  $O(1)$ ith child, parent, subtree size