Level ancestor problem

6.897
L19.2
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[Berkman & Vishkin – JCSS 1994;
Dietz – WADS 1991; Alstrup & Holm – ICALP 2000;
TODAY ⊛ Bender & Farach-Colton – TCS 2004]

↖ dynamic tree

- preprocess static rooted tree
- level-ancestor query: find ℓ^{th} ancestor of node v
  ≡ find depth-d ancestor of node v
  ($d + \ell = \text{depth}(v)$; store depth of each node)
- goal: $O(n)$ preprocessing/space
  $O(1)$ query

Solutions: [Bender & Farach-Colton – TCS 2004]

① lookup table on all possible queries:
- $O(n^2)$ space
- $O(1)$ queries

② jump pointers:
- each node stores 1ST, 2ND, 4TH, 8TH, ..., $(2^i)^{TH}$, ... ancestor
⇒ $O(n \lg n)$ space
- query: $v \leftarrow (\lfloor \ell \rfloor = 2^{\lfloor \lg \ell \rfloor})^{th}$ ancestor of v
  $\ell \leftarrow \ell - \lfloor \ell \rfloor$
  repeat
- $\ell/2 < \lfloor \ell \rfloor \leq \ell$
⇒ each step reduces ℓ by at least half
⇒ $O(\lg n)$ query