6.897
L 17.3
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$t_q = \Omega(\lg_B n)$ lower bound for existential queries where $B = t_u \lg^2 n$

- update nodes level by level as before
- state u_i of node i = marked with probability $p = \frac{1}{\lg n}$

$\Rightarrow E[\#\text{marked ancestors of a fixed leaf}] = \frac{1}{\lg n} \cdot \lg_B n = \frac{1}{\lg B} = o(1)$.

- construct $\overline{R}_j \supseteq R_j$ iteratively for $j = 1, \ldots, \lg_B n$
  - $[u]_j = \{w \in \{0,1\}^n \mid u \& w$ differ only on j^{th} level and $\overline{R}_{<j}(u) = \overline{R}_{<j}(w)\}$
    - (later will show that $|[u]_j|$ is large enough: for $\Omega(B^j)$ i's, $\exists a, b \in [u]_j$ s.t. $a_i = 1$ & $b_i = 0$) ⊛
  - find a fooling set $F_j(u) \subseteq [u]_j$ with $|F_j(u)| = (\lg n) = 1/p$ such that ⊛ is true on $F_j(u)$
    - existence (sketch): pick such a set randomly works w.h.p. for suitable (small) const in ⊛'s $\Omega(B^j)$
  - $\overline{R}_j(u) = \left[\bigcup_{\substack{w \in F_j(u) \\ \text{or } w = u}} R_j(w)\right] \setminus \overline{R}_{<j}(u)$
- properties of $\overline{R}$ family:
  - $\overline{R}_j(u) \cap \overline{R}_k(u) = \emptyset \quad \forall j = k$ (because of subtraction)
  - $w \in F_j(u) \Rightarrow$ executions on u & w differ only in cells in $\overline{R}_j(u)$
    - consider cell in $\overline{R}_i(u)$, $i < j$
      $\Rightarrow$ same because everything in $\overline{R}_{<j}(u)$ is same as in $\overline{R}_{<j}(w)$.
    - cell in $\overline{R}_j(u)$ allowed to differ
    - cell in $\overline{R}_j(w) = \left[\bigcup_{v \in F_j(w) \cup \{w\}} R_j(v)\right] \setminus \overline{R}_{<j}(w)$
      $= F_j(u)$ because $[u]_j = [w]_j$; $= \overline{R}_{<j}(u)$
      $\subseteq \overline{R}_j(u)$
    - cell in $\overline{R}_i(u)$, $i > j$ in $\overline{R}_\ell(w) \Rightarrow \ell > j$
      $\rightarrow$ past is equal for u & w.
- argue query reads something in $\overline{R}_j$ with prob. $\Omega(1)$