- let R_j = cells written during epoch j
  ["register", not "read"]
- for random u, looking at $R_{<j} = \bigcup_{i=1}^{j-1} R_i$
  can only tell us state of $o(B^j)$ nodes
- counting query needs state of a random ancestor on level j
  ⇒ with probability $\Omega(1)$, read some cell from level j

another view:

- $[u]_j = \{ w \in \{0,1\}^n \mid$ when algorithm is run on u & w, all differing cells are in $R_j \}$
- then $|[u]_j| = 2^{-\Omega(B^j)}$

Existential queries

- lower bound also holds in nondeterministic (query) setting
  - prover reveals t_q cells
  - verifier must be convinced of answer
- for existential queries, ∃ easy nondet. query alg.
  if there is a marked ancestor ← exploit this

$\Omega(\sqrt{\lg n})$ lower bound for existential queries

- suppose $t_q = t_u = O(\sqrt{\lg n})$ for existential queries
- construct a nondeterministic alg. for counting queries
- build trees with $B = 2^{\sqrt{\lg n}} = \Omega(\lg_B n) = \Omega(\sqrt{\lg n})$
  ← slack here is OK in previous lower bound
  ⇒ height $h = \sqrt{\lg n}$
- maintain each of 2^h trees with existential DS.
  → for each subset S of levels [2^h choices]
  flip status of nodes in level in S
  ⇒ update = $2^h \cdot t_u$
- query: - guess marked ancestors ~ say on levels T
  - verify all marked & all others unmarked → query in $S=T$ ⇒ FALSE
  - return answer
- guess correct ⇔ in tree with $S=T$, all nodes in path are unmarked