Lower bounds for marked ancestor

6.897
L17.1
Apr. 7, 2005

- Chronogram technique [Fredman & Saks 1989]
- variations of query:
  - existential: is there a marked ancestor of v?
  - counting: # marked ancestors → harder than existential → easier than lowest
- t_u = update time, t_q = query time

Theorem: for a perfect tree with branching factor $B = t_u \cdot \lg^2 n$,

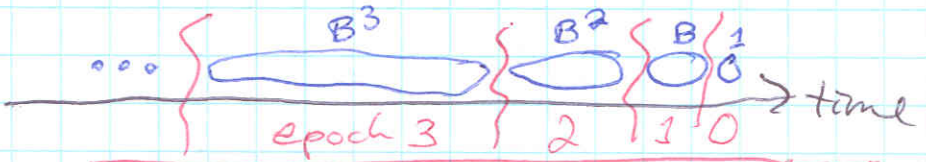
$$t_q = \Omega(\log_B n) = \Omega\left(\frac{\lg n}{\lg t_u + \lg\lg n}\right)$$ ← tradeoff

↳ here, worst case ~ amortized possible

Operation sequence:

- for each level of tree from bottom up
  - update all nodes on the level
    - ↳ make node marked with probability $1/2$] EPOCH
- query a random leaf

only make marks! ⇒ incremental connectivity, worst case



larger epochs are older

→ for counting, or counting modulo 2 (WARMUP)

Claim: query needs $\Omega(1)$ cells from each epoch, in expectation

$\Rightarrow t_q = \Omega(\lg_B n)$ [height of tree]

- state $u \in \{0,1\}^n$ of all nodes is uniform.
- suppose no cell read from epoch (level) j
- give algorithm all cells from epochs $k \neq j$
  - $k > j$ ⇒ useless (don't know future in past)
  - $k < j$ ⇒ only $t_u \sum_{i=1}^{j-1} B^i = t_u \cdot O(B^{j-1})$ such epochs
- epoch j has B^j bits of information

⇒ cells from later epochs give $O(t_u \lg n \, B^{j-1})$ bits

$= O\left(\frac{B^j}{\lg n}\right) = o(B^j)$