6.897
L16.1
Apr. 5, 2005

Tree decompositions

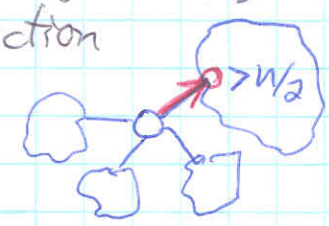
(-preferred paths) → saw in Tango trees & link-cut trees
- heavy-light decomp. → saw in link-cut trees
- separator decomp. } → TODAY
- ART decomp. }

Separator theorem on trees

[Jordan – "Sur les assemblages de lignes" – Journal Reine Angew. Math. – 1869]

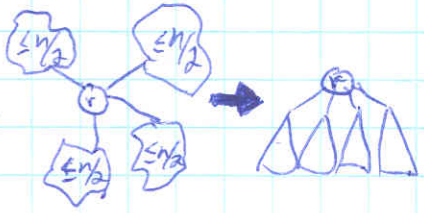
Given any tree on n vertices, there is a vertex whose removal disconnects the tree into components each of $\leq n/2$ vertices.

Proof:
- pick a vertex
- if its removal leaves a piece with $> n/2$ vertices then walk one step in that (unique) direction
- new component of old vertex + other pieces has $< n/2$ vertices



⇒ never go back to old vertex
⇒ terminate. □

Separator decomposition

- apply separator theorem → root of new tree
- recurse on components → children of root

⇒ depth of new tree is $O(\lg n)$