6.897
L15.1
Mar. 31, 2005

Range minimum queries (RMQ)

- array of n numbers ~ static
- query: min in range $A[i], \ldots, A[j]$
- Cartesian tree [Gabow, Bentley, Tarjan – STOC 1984]
  - root = min in array, say $A[i]$
  - left subtree: recurse on $A[1], \ldots, A[i-1]$
  - right subtree: recurse on $A[i+1], \ldots, A[n]$
  - $O(n)$ time construction

} saw in L13.2 (fast sorting)

$\Rightarrow$ RMQ(i,j) = LCA(i,j)
(array indices) (tree nodes)

Lowest common ancestor (LCA)

- tree on n nodes ~ static
- query: LCA of two given nodes
- $O(1)$ time/query, $O(n)$ space [Harel & Tarjan – SICOMP 1984]
- simplification [Bender & Farach-Colton LATIN 2000] * TODAY
  - PRAM basis: [Berkman et al. – STOC 1989]
- dynamization [Cole & Hariharan – SICOMP to appear; SODA 1999]
  - insert/delete leaves
  - subdivide/merge edge
  - $O(1)$/op.