New structure [Demaine & Pătraşcu – this week]

6.897
L14.2
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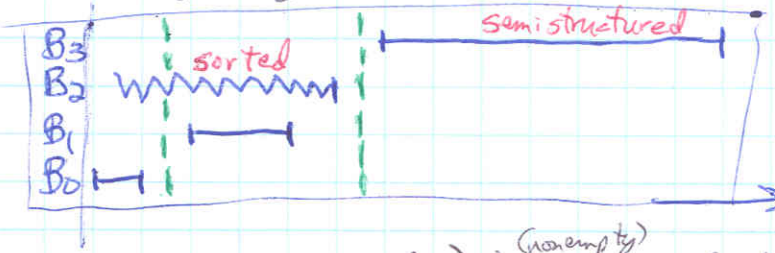
- $\Theta(\lg n)$ buckets $B_0, B_1, B_2, \ldots$
- B_i either
  - **empty**
  - **sorted** $\Rightarrow \le 2^i$ elements, in linked list (↳ charge)
  - **semistructured**:
    - unstructured part U_i of arbitrary size (unsorted linked list) (↳ charge)
    - structured part S_i: $\Omega(2^{i/2})$ groups of size $\Theta(2^{i/2})$
      ↳ groups in order, but each group unsorted
      (↳ charge for additional groups beyond $\Theta(2^{i/2})$)

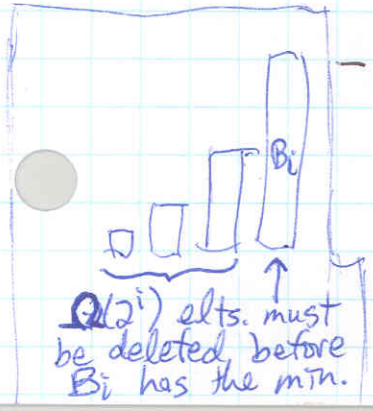
(– B_0 always semistructured)

- $i<j \Rightarrow$ elts. in $B_i <$ elts. in B_j if you ignore all but max elt. in sorted B_i's



- **atomic heap** on $\min(B_i)$ $\forall i$ (nonempty) to find which contains min
- atomic heap on $\min(B_i)$ $\forall$ semistructured B_i to find where to insert new element
- decrease-key removes from existing bucket & calls insert (but essential that it's only moving left)
- delete-min
  - find lowest nonempty bucket B_i
  - if B_i sorted: extract from list
  - if B_i semistructured:
    - if $|B_i| = O(2^i)$ then sort & turn into sorted bucket $\Rightarrow \Theta(2^i)$
    - if $|B_i| = \omega(2^i)$ then
      - for $j = i, i+1, \ldots$ ↝ exponential blastoff
        - put elts. of B_i into bag
        - drop $\Theta(2^j)$ smallest elts. from bag to B_i
        - stop when bag is empty

(computed after sizes determined via recursive median cutting)



- structured parts can be split/merged in $O(2^{i/2})$ time
- structured part can be built from unsorted in $O(2^i)$ time [Han & Thorup–FOCS 2002]