6.897
L14.1
Mar. 31, 2005

Priority queues: Insert, Delete, Find-Min

Relation to graph algorithms

- shortest paths via Dijkstra [Num. Math. 1959]
  - $O(m)$ decrease-key
  - $O(n)$ delete-min
  - monotone: min never decreases
- minimum spanning tree via Prim [Bell Sys. Tech. J. 1957]
  - $O(m)$ decrease-key
  - $O(n)$ delete-min
  - not monotone
- directed MST via Edmonds [J. Res. Nat. Bur. Standards 1967]
  - main additional op: meld — merge 2 queues

Results

- heap — $O(\lg n)$/op.
- Fibonacci heap [Fredman & Tarjan — JACM 1987]
  - $O(1)$ decrease-key & insert } amortized
  - $O(\lg n)$ delete-min
  - $\Rightarrow O(n \lg n + m)$ for Dijkstra, MST
- worst-case version [Brodal — SODA 1996]
- atomic heap [Fredman & Willard — JCSS 1994]
  - $O(1)$ am. time for $O(\text{poly} \lg n)$ elts.
  - essentially a fusion-tree node + lookup tables to dynamize
  - $\Rightarrow O(n+m)$-time alg. for MST [same paper]
    - also possible randomized in comparison model [Karger, Klein, Tarjan — JACM 1995]
    - $O(m \alpha(m,n))$ deterministic in comparison [Chazelle — JACM 2000]
- $O(n S(n,w))$ sorting $\Rightarrow O(S(n,w))$ am./op. monotone priority queue [Thorup — JComp 2000]
  - $\Rightarrow O(S(n,w))$ w.c./op. priority queue [Thorup — FOCS 2002]
- $O(P(n,w))$/op. priority queue $\Rightarrow O(P(n,w)\,\alpha(n))$/op. meldable priority Q. [Mendelson, Thorup, Zwick — SODA 2004]

essentially removed by [Mendelson, Tarjan, Thorup, Zwick — SWAT 2004]

- Conjecture/almost-theorem: [Demaine & Pătraşcu — this week]
  $O(n S(n,w))$ sorting $\Rightarrow$ $O(S(n,w))$ delete-min, $O(1)$ decrease-key & insert } amortized