Integer search problems

6.897
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Predecessor, dynamic:

- $O(\min\{\lg_w n, \frac{\lg w}{\lg\lg w}\lg\lg n, \frac{\lg w}{\lg \frac{a}{\lg n}}\lg\lg n\})$
  [Raman – TR 1995] ← expected
- OPEN: can you beat vEB i.e. $o(\lg w)$?
- Conjecture: if dynamic, or static & space $= O(n \operatorname{poly} w)$ ($\Rightarrow$ space $= O(n)$ by indirection)
  (almost theorem) then $\Omega(\lg w)$

Dictionary:

- randomized $O(1)$
- deterministic: update $O(\operatorname{poly}\lg n)$
  query $O(\operatorname{poly}\lg\lg n)$ [Pagh – SWAT 2000]
  - OPEN: $O(\lg w)$ [vEB] or $O(\lg_w n)$? [fusion]

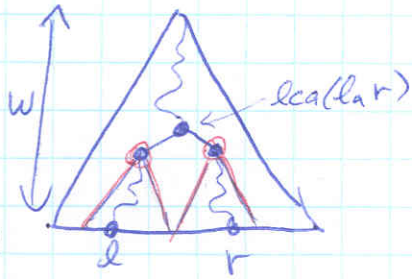
Range reporting: find any elt. in range $[l, r]$

($\Rightarrow$ can follow linked list & navigate k in $+O(k)$ time)

- static: $O(nw)$ space, $O(1)$ query [Miltersen, Nisan, Safra, Wigderson – JCSS 1998]
- static: $O(n)$ space, $O(1)$ query, optimal [Alstrup, Brodal, Rauhe – STOC 2001]
- dynamic: $O(\lg w)$ update [Mortensen, Pagh, Pătraşcu – STOC 2005]
  !! $O(\lg\lg w)$ query
  $O(n)$ space
  + trade-off

$O(nw)$-space static range reporting

- query: – find lca of l & r
  – find max of left child subtree & min of right child subtree
  – one must be in $[l, r]$ if any are



- DS stores min & max of each subtree
  $\Rightarrow O(nw)$ space ~ each elt. appears in w subtrees
- can't use indirection: interval might miss all representatives