More predecessor bounds: (upper & lower!)

w vs. n vs. space tradeoff

6.897
L12.1
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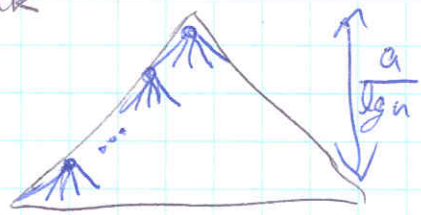
Recall: lower bounds from last class

$-\Omega(\min\{\underbrace{\lg_{b=w} n}_{\text{matched by fusion trees} \Rightarrow \text{tight}}, \underbrace{\lg_a w}_{\text{vEB only } \lg w \text{ can we improve UB?}}\})$

$-\forall w \exists n: \Omega\left(\frac{\lg w}{\lg\lg w}\right)$ $\left[\& \forall n \exists w: \Omega\left(\sqrt{\frac{\lg n}{\lg\lg n}}\right)\right]$

$O\left(\frac{\lg w}{\lg \frac{a}{\lg n}}\right)$ data structure: [Pătraşcu & Thorup – manuscript 2005]

- break query word into $\Theta\left(\frac{a}{\lg n}\right)$ equal-size chunks
- send $\Theta(\lg n)$-bit hash of each chunk
  $\Rightarrow \Theta(a)$ bits total ✓
- in height-$\Theta\left(\frac{a}{\lg n}\right)$ tree of chunks,
  branching factor $= O\left(2^{w\frac{\lg n}{a}}\right)$
  but effectively branching factor must be $\le n$



$\Rightarrow$ w.h.p. each hash is correct
$\Rightarrow$ find first chunk where query falls off "correct paths" via one probe to Bob
$\Rightarrow$ reduce problem to that chunk
$w \searrow w / \frac{a}{\lg n}$

- $O\left(\frac{\lg w}{\lg \frac{a}{\lg n}}\right)$ optimal for $a = \Omega(\lg^{1+\varepsilon} n)$
- space: $2^{\Theta(a)}$ words for Bob's lookup table
  $\times$ $n \cdot t$ different subsets of elts. considered
  (each element appears in $\le t$ subsets)
  ↳ predecessor time
  $= 2^{\Theta(a)} \cdot n \cdot t$
  $\le 2^a$ for appropriate constants & $a = w(\lg n)$