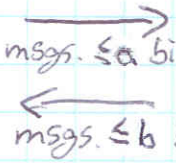
Colored predecessor:

- each element has a color of red or blue
- predecessor/successor just needs to report color
- easier problem ⇒ LB stronger
- useful for reductions later

Communication complexity viewpoint:

Alice x — Bob y — $f(x,y)$



- Alice knows input x
  - ↳ query algorithm
  - ↳ query
- Bob knows input y
  - ↳ DS/memory
  - ↳ contents of DS/memory
- a = #address bits $= O(\lg n)$ if space $= n^{O(1)}$ [typical assumption]
- b = word size w
- # messages $= 2 \cdot$ # cell probes

Predecessor lower bound:

$$\Omega\left(\min\left\{\lg_a w, \lg_b n\right\}\right)$$

Beame-Fich-Xiao bound: ($a = O(\lg n)$)

- lower bound smallest (worst) when w & n related by

$$\lg_a w = \lg_b n$$

$$\text{i.e. } \frac{\lg w}{\lg\lg n} = \frac{\lg n}{\lg w} \quad (\Theta)$$

$$\text{i.e. } \lg^2 w = \lg n \lg\lg n \quad (\Theta)$$

$$\text{i.e. } \lg w = \sqrt{\lg n \lg\lg n} \quad (\Theta)$$

$$\Rightarrow \lg\lg n = \lg\lg w \quad (\Theta)$$

$$\Rightarrow \text{LB} = \frac{\lg w}{\lg\lg n} = \sqrt{\frac{\lg n}{\lg\lg n}} \text{ and } \frac{\lg w}{\lg\lg w}$$