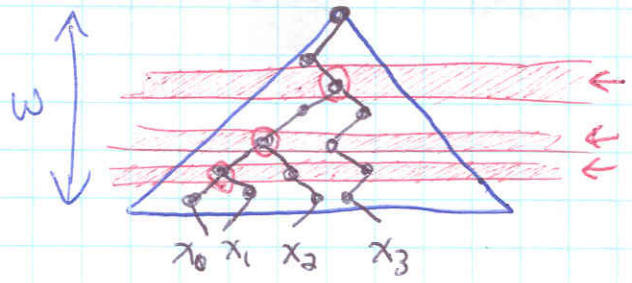
Distinguishing $k = O(w^{1/5})$ keys

6.897
L10.2
Mar. 8, 2005

- view keys $x_0, x_1, \ldots, x_{k-1}$ as binary strings (0/1)
  i.e. paths from root of binary tree (left/right)



- # branching nodes $= k-1$

$\Rightarrow$ # levels containing branching nodes $\leq k-1$

$=$ # bits where $x_0, x_1, \ldots, x_{k-1}$ first differ (not identical prefix)

- call these important bits $b_0 < b_1 < \cdots < b_r$ $(r < k = O(w^{1/5}))$

[perfect] Sketch(x) = extract bits $b_0, b_1, \ldots, b_{r-1}$ from word x
i.e. r-bit vector with ith entry = b_ith bit of x

$\Rightarrow$ sketch$(x_0) < \cdots <$ sketch(x_{k-1}) are all different & fit (packable) in a word: $kr = O(w^{2/5})$ bits

- computable in $O(1)$ time as an AC^0 operation
  [Andersson, Miltersen, Thorup – TCS 1999]

Idea: in parallel, compare sketch(q), q = query, to sketch$(x_0), \ldots,$ sketch(x_{k-1})

- AC^0 operation on $O(1)$ words

$\Rightarrow$ find where sketch(q) fits (predecessor/successor) among sketch$(x_0) < \cdots <$ sketch(x_{k-1})