Fusion trees [Fredman & Willard 1993]

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$O(\lg_w n)$ time / predecessor & successor
[$O(\lg_w n + \lg\lg n)$ time/update] ← Problem 5
& [Andersson & Thorup – 2002 arXiv:cs.DS/0210006]
– here focus on static case
$O(n)$ space

Top-level idea:

– B-tree with branching factor $\Theta(w^{1/5})$

⇒ height
$= \Theta(\lg_w n) = \Theta\left(\frac{\lg n}{\lg w}\right)$

– to search in $\Theta(\lg_w n)$, in $O(1)$ time
need to find correct branch at each node
– not enough time to read node ($\Theta(w^{1/5})$ words)
or $\Theta(\lg$ (node size)) etc.

Fusion-tree node:

given k keys $x_0 < x_1 < \cdots < x_{k-1}$, $k = O(w^{1/5})$,
preprocess in $k^{O(1)}$ time
subject to predecessor/successor in $O(1)$ time