6.897
L 9.3
Mar. 12, 2003

Key-independent optimality [Iacono-ISAAC2002]

- suppose actual key values & ranks were "meaningless"
- might as well take a uniformly random bijection b on the keys
- $E[\text{dynamic OPT}(b(\text{seq.}))] =: \text{key-indep. dynamic OPT}(\text{seq.})$

Thm: key-indep. dynamic $OPT(\text{seq.}) = \Theta\left(\sum_{i=1}^{m} \lg w_i\right)$

where w_i = # distinct elements accessed since last access to x_i
= working-set number

⇒ splay trees & working-set structure are key-indep. OPT imd

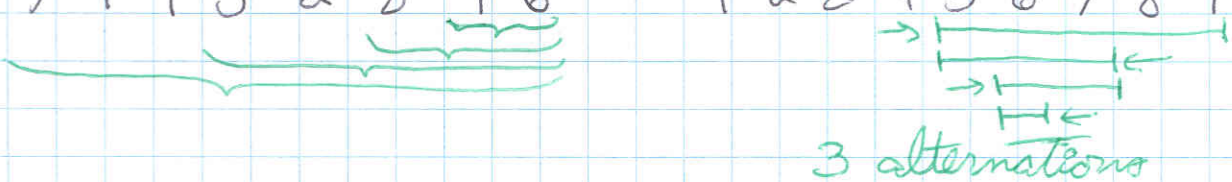
«proof uses:»

Wilbur's (second) lower bound [FOCS 1986]

- request sequence $x_1, x_2, \ldots, x_m$
- consider amortized cost of x_j
  - look at where x_j fits among $x_{j-1}, \ldots, x_1$ for $i = j-1, j-2, \ldots, 1$
    say between a_i and b_i.
  - as i decreases & range of x's grows,
    a_i will increase and b_i will decrease.
  - amortized cost(x_j) ≥ # alternations between a_i increasing and b_i decreasing.
- total cost of sequence ≥ sum of these costs.
  ↳ in binary search tree & rotation model

E.g.: max. LB for 9 requests

3 7 1 9 5 2 8 4 6 1 2 3 4 5 6 7 8 9



3 alternations

Total: 22 alternations

OPEN: is dynamic OPT = O(Wilbur's lower bound)?