Analysis of Add

6.897
L9.2
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- potential $\Phi = c \times$ (# extreme children i.e. # children in first 1/4 or last 1/4 of parent)

Case 1: $O(2^l)$ time, $O(1)$ potential gain ($O(1)$ per level)

Case 2: How do z's children evolve, z in T_{i+1}?

- when z added, just z as child.
- each new child is within $2 \times 2^{2^i}$ of a previous child

$\Rightarrow$ to get to new "child" outside $2^{2^{i+1}}$ of z, must have added $\geq \frac{2^{2^{i+1}}}{2 \times 2^{2^i}}$ children beforehand.

& $\geq \frac{1}{2} \cdot \frac{2^{2^{i+1}}}{2 \times 2^{2^i}}$ in extreme 1/4 of z's children.

$\Rightarrow$ potential loss of $\geq c \times \frac{1}{2} \cdot \frac{2^{2^{i+1}}}{2 \times 2^{2^i}} = c \times \frac{1}{4} 2^{2^i}$

whereas cost of Add $\rightarrow T_{i+1}$ is $O(2^{i+1})$ which is dominated for c sufficiently large.

0 amortized cost (but then a Case 1)

$\Rightarrow$ overall amortized cost $= O(2^l)$.