6.897
L 8.5
Mar. 10, 2003

**Add** $x \to T_i$ with same parent as $y$

**Case 1:** if $x$ is within $2 \cdot 2^{i+1}$ of parent($y$) then we can add it directly:

insert:
- insert $x \to T_i$, enqueue $x \to Q_i$
- link $x \leftrightarrow$ parent($y$), maybe remove parent($y$) $\leftarrow Q_{i+1}$

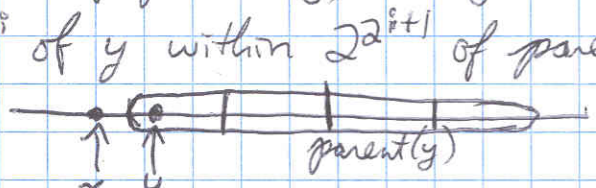
balance:
- dequeue $z$ from $Q_i$, delete $z \leftarrow T_i$
- unlink $z \leftrightarrow$ parent($z$), maybe enqueue parent($z$) $\to Q_{i+1}$

$O(2^i)$ time.

L 9.1

**Case 2:** $x$ not within $2 \cdot 2^{i+1}$ of parent($y$); say $x \ll$ parent($y$)

**but:** $x$ is within $2 \times 2^{2^i}$ of $y$ within $2 \cdot 2^{2^{i+1}}$ of parent($y$).




- Add $z =$ parent($y$) $- 2 \cdot 2^{2^{i+1}}$ (extreme of parent($y$)'s range) $\to T_{i+1}$ with same parent as parent($y$) (recursive)
- move left 1/4 of parent($y$)'s children to third 1/4 of $z$'s children (& remove $z$ from $Q_{i+1}$, maybe enqueue parent($y$) $\to Q_{i+1}$)
- Add $x \to T_i$ with parent $= w$ ~ Case 1 $\Rightarrow$ cheap.