How far do we have to look?

6.897
L8.4
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- if x is within $2\times 2^{2^i}$ of y in T_i
  then x is within $2\times 2^{2^{i+1}}$ of parent(y) in T_{i+1}
  Proof: x within $2\times 2^{2^i}$ of y within $2^{2^{i+1}}$ of parent(y)
  $\Rightarrow$ x within $2\times 2^{2^i} + 2^{2^{i+1}} < 2\times 2^{2^{i+1}}$ of parent(y) □

- in particular, right after x is accessed,
  x is within $2\times 2^{2^i}$ of parent$^{(i-1)}$(x in T_1) in T_i.

- in fact, any elt. within $2^{2^i} - 2\times 2^{2^{i-1}}$ of x
  is also within $2\times 2^{2^i}$ of parent$^{(i-1)}$(x in T_1) in T_i.
  (how long does this survive in T_i?)

- parent$^{(i-1)}$(x in T_1) in T_i only removed after $\geq 2^{2^i} - 2^{2^{i-1}}$ dequeues/queue deletes
  - starts with a child $\Rightarrow$ not in queue
  - so when it enters queue, enters at end
  - queue has $\geq 2^{2^i} - 2^{2^{i-1}}$ elements

- ≤ 2 dequeues/queue deletes per queue per Search

$\Rightarrow \geq \frac{2^{2^i} - 2^{2^{i-1}}}{2}$ Searches before parent$^{(i-1)}$(x in T_1) in T_i is removed from T_i.

$\Rightarrow$ if we search for elt. y within time $\frac{2^{2^i} - 2^{2^{i-1}}}{2}$ & within space $2^{2^i} - 2^{2^{i-1}}$ } unified number: $\Theta(2^{2^i})$

of x, then y is within $2\times 2^{2^i}$ of parent$^{(i-1)}$(x in T_1) in T_i

$\Rightarrow \ell \leq i \Rightarrow$ cost $= O(2^i) = O(\lg 2^{2^i})$.

$\Rightarrow$ unified.