Unified property [Iacono – SODA 2001]

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L8.1
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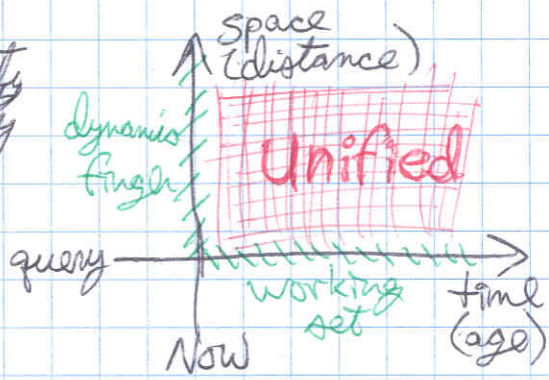
Motivating examples

	Dynamic finger	Working set	Unified
$1\ 2\ \cdots\ n\ \ 1\ 2\ \cdots\ n\ \cdots\ 1\ 2\ \cdots\ n$	$O(m)$	$O(m \lg n)$	$O(m)$
$1\ 2\ \cdots\ n\ \ 1\ \ n\ \ 1\ \ n-1\ \cdots\ 1\ \ n$	$O(m \lg n)$	$O(m)$	$O(m)$
$1\ \ n/2\ \ 2\ \ n/2+1\ \ 3\ \ n/2+2\ \cdots\ n/2-1\ \ n$	$O(m \lg n)$	$O(m \lg n)$	$O(m)$

Intuition:
- Dynamic finger captures spatial locality
- Working set captures temporal locality
- Unified captures both



Unified property: Suppose request sequence is $x_1, x_2, \ldots, x_m$.

$$\text{amortized cost}(x_i) \le \lg \min_{j \le i} \left(1 + \underbrace{|x_i - x_j|}_{\text{spatial (rank)}} + \underbrace{\quad\quad}_{\text{temporal}}\right)$$

where t_i = # distinct elements accessed up to x_i.

OPEN: Do splay trees satisfy the unified property? ("unified conjecture")

OPEN: Does any dynamic search tree have the unified property?

THM: "Unified structure": comparison data structure with unified property.

OPEN: simpler (practical?) structure — currently 21 pointers/node (plus 1 int per node)

OPEN: dynamic unified — support Insert & Delete