Sketch of splay-tree analysis

6.897
L7.4
Mar. 5, 2003

- suppose we assign arbitrary weights $w(i)$ to each element i.
- size of node x is $s(x) = \sum\{w(i) : i \text{ in subtree rooted @ } x\}$.
- rank of node x is $r(x) = \lg s(x)$
- Access theorem: If splay tree currently has root t, amortized cost to splay node x is
$$3(r(t) - r(x) + 1) = O(\lg \frac{s(t)}{s(x)}).$$

Proof uses potential $\Phi = \sum_{\text{nodes } x} r(x)$. Assumes all elements accessed.

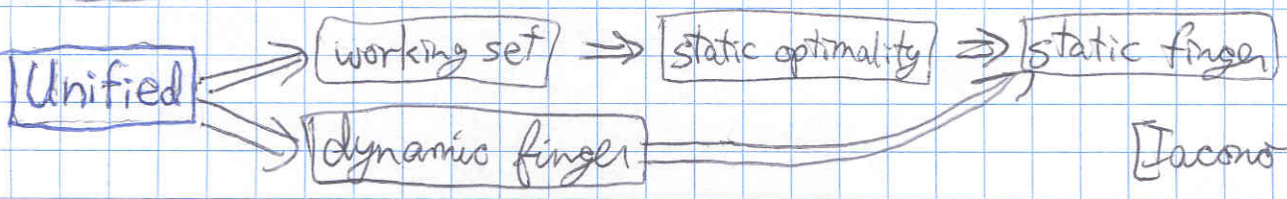
CONSEQUENCES:

- lg n bound: $w(i) = 1 \ \forall i \Rightarrow s(t) = n \Rightarrow O(\lg \frac{n}{\geq 1}) = O(\lg n)$
- static optimality: $w(i) = p_i = f_i/m \Rightarrow s(t) = 1$
access to $x \Rightarrow s(x) \geq w(x) \Rightarrow O(\lg \frac{1}{\geq w(x)}) = O(\lg 1/p_x)$
Total cost: $O(m \sum_{i=1}^{n} p_i \lg 1/p_i) = O(mH)$ ENTROPY
- static finger theorem: fix your "finger" on a node f.
$w(i) = \frac{1}{(1+|i-f|)^2} \Rightarrow s(t) \leq \sum_{k=-\infty}^{+\infty} \frac{1}{k^2} = \pi^2/3$
$\Rightarrow$ cost $= O(\lg \frac{O(1)}{w(i)}) = O(\lg(1+i-f)^2) = O(\lg(1+|i-f|))$.

MORE:

- working set theorem: amortized cost of accessing an element is $O(\lg(\#\text{distinct elements since last access}))$ [initially $\lg n$]
In particular, if use only k elts, cost is $O(\lg k)$.
- scanning theorem [Tarjan–Combinatorica 1985]:
splaying all elements one-by-one in order costs $O(n)$ from any tree
- dynamic finger theorem: [Cole 1995]:
if you just accessed element x, then accessing y costs
$O(\lg(1+|x-y|))$.

IMPLICATIONS:



[Iacono 2000]