Self-adjusting trees (unknown distribution)

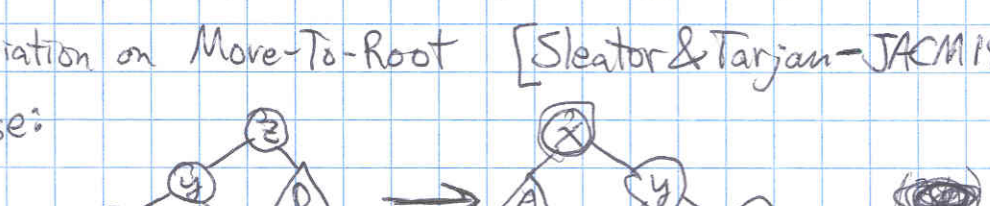
6.897
L 7.3
Mar. 5, 2003

- Transpose analog: [Allen & Munro - JACM 1978]
  - upon request, rotate 1 closer to root

  - bad: for uniform searches, all binary trees generated with same probability
    ⇒ average search cost is $\Theta(\sqrt{n})$ ∴

- Move-To-Root by repeated rotation: not bad
  - within a factor of $2 \ln 2 \approx 1.38$ of (static) OPT tree in stochastic model
  - but not statically optimal ~ many nodes get deeper

- Splay trees: variation on Move-To-Root [Sleator & Tarjan - JACM 1985]
  - Zig-Zig case:



(analogous to Rotate(z); Rotate(y))

  - Zig-Zag case:

(analogous to Rotate(x); Rotate(z))

  - at the end, may need a single rotation

Results:
- $O(\lg n)$ amortized time/op. (search as well as Insert & Delete)
- statically optimal ⇒ entropy bound
- BIG CONJECTURE: dynamically optimal