6.897
L 7.1
Mar. 5, 2003

Theorem: amortized cost of ONR to access element $\leq 1 + 4\lceil \lg r \rceil$
where r = # distinct elements accessed since this element last accessed, including it

Corollary: total cost $\leq m + 4\sum_{i=1}^{n} \lceil \lg m/f_i \rceil$ $\quad f_i$ = # occurrences of i

amortized cost for elt. i $= O(\lg 1/p_i)$ $\quad p_i = f_i/m$.

"expected" cost for random elt. $= O\left(\sum_{i=1}^{n} p_i \lg 1/p_i\right)$ → ENTROPY

total cost $= O\left(m \sum_{i=1}^{n} p_i \lg 1/p_i\right)$

Proof of theorem:

Bad idea 1: charge to last block (elements in) → those elements might (most of) never be requested!

Bad idea 2: charge to [redacted] front elements → not enough!

Soak the middle class:

charge cost of ρ to penultimate block b of size $\frac{\rho+1}{4}$
(unless request 1ST element ⇒ charge to it)

Claim: an element i is charged at most once in any block between two requests for i.

Case 1: i gets charged in block b & stays in b or moves forward

- Elements from new position of i to end of block $b+1$ serve as buffer — remain in same relative order until i accessed, because i will be 1ST accessed.
- i can't be charged unless moved to block $b+1$
  → where buffer might not fill block $b+2$ & then $b+2$ element requested
  → then in Case 2.

Case 2: i moves to block $b+1$

- may get charged here by request in block $b+2$.
  But again only once, etc.

≤ 1 charge per block.

If r distinct elements accessed between two accesses to i
then i maximally in position r ⇒ ≤ $\lceil \lg r \rceil$ charges
⇒ ≤ $4\lceil \lg r \rceil$ cost em.