Questioning the model [Munro – ESA 2000]

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L6.3
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- How much "should" it cost to reverse first $n/2$ elts?

$$1 \quad 2 \quad \cdots \quad n/2 \quad n/2+1 \quad n/2+2 \quad \cdots \quad n$$
$$\Rightarrow n/2 \quad n/2-1 \quad \cdots \quad 1 \quad n/2+1 \quad n/2+2 \quad \cdots \quad n$$

- Just 1 pointer memory — $\Theta(n^2)$ adjacent swaps (Sleator & Tarjan)
- $O(1)$-pointer memory — $\Theta(1)$ pointer changes
  $\Theta(n)$ to scan the elements & find $n/2$.
  ← (Munro)

Order-by-Next-Request (ONR)

- offline (omniscient) algorithm
- upon search for element in position i:
  - continue scan to position $\lceil\lceil i \rceil\rceil := 2^{\lceil \lg i \rceil}$. ← hyperceiling
  - reorder (sort) these elements according to when next requested.
    → charge $\lceil\lceil i \rceil\rceil = \Theta(i)$.

E.g.:

Search	Cost	List
1	–	1 2 3 4 5 6 7 8 9 10 ...
2	2	2 1 –
3	4	3 4 2 1 ...
4	2	4 3 ...
5	8	5 6 7 8 1 2 3 4 ...
6	2	6 5 – –
7	4	7 8 6 5 – –
8	2	8 7 ...
9	16	9 10 11 12 13 14 15 16 1 2 3 ...
...	...	...

Total cost for permutation: $n(\frac{1}{2}\cdot 2 + \frac{1}{4}\cdot 4 + \frac{1}{8}\cdot 8 + \cdots) \sim n \lg n$

In contrast: MTF costs n^2 (with startup model or nasty permutation matching initial order)
(or any online algorithm)

Sort i elements in $\Theta(i)$ time?

(Counting sort is cheating – supposed to use comparison model.)

Invariant: block of elements @ positions $2^j+1, 2^j+2, \ldots, 2^{j+1}-1$ is sorted, $\forall j$.

$\Rightarrow$ merge $\lceil \lg i \rceil + 1$ blocks of length $2^0, 2^1, 2^2, \ldots, 2^{\lceil \lg i \rceil}$

Cost: $\Theta(2^1 + 2^2 + 2^3 + \cdots + 2^{\lceil \lg i \rceil}) = \Theta(i)$.