MTF is 2-competitive

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Potential function Φ = # inversions between MTF list and OPT list as we run them on same input in parallel

i.e. # pairs (i,j) where i preceding j is reversed in MTF list vs. OPT list.

Free swaps made by OPT decrease Φ ($\rightarrow$ closer to MTF)
$\Rightarrow$ "free rebate" on Φ.

Paid swaps might increase Φ by 1 each
– charge this increase to swap cost by OPT.

Cost of searching for i
– suppose i is in position k_{OPT} in OPT list & k_{MTF} in MTF list
– suppose out of $k_{MTF}-1$ elements in front of i in MTF list, there are f elements in front of i in OPT list & b elements behind i in OPT list.
$\Rightarrow f+b = k_{MTF}-1$.

– after MTF, all $k_{MTF}-1$ of these elements move behind i in MTF list
$\Rightarrow \Phi$ increases by $f-b$.

– total amortized cost of MTF $= \underbrace{k_{MTF}}_{\text{search}} + \underbrace{(f-b)}_{\text{increased potential}}$

$$= (f+b+1) + (f-b)$$
$$= 2f+1$$
$$\leq 2(k_{OPT}-1)+1$$
$$= 2\underbrace{k_{OPT}}_{\text{comparison-cost of OPT}} - 1$$

$\therefore$ cost(MTF) $\leq 2 \cdot$ comparison-cost(OPT) + paid-swaps(OPT) − free-swaps(OPT) − m
$< 2 \cdot$ total-cost(OPT). □