Reducing space in vEB

6.897
L 2.2
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- store sub[S] as dynamic hashtable & store only nonempty substructures

BLACK BOX: dynamic perfect hashing — $O(n)$ space
— $O(1)$ time Insert/Delete/Search w.h.p.

- charge space for hashtable to nonempty substructures, each of which stores an elt. as min
- charge each summary elt. to these elts. as well

$\Rightarrow O(1)$ space / nonempty substructure which directly stores 1 elt.

$\Rightarrow O(n)$ space.

Dynamic perfect hashing [Dietzfelbinger, Karlin, Mehlhorn, Meyer auf der Heide, Rohnert, Tarjan 1990]

Universal set H of hash functions: $\mathcal{U} \to \{0, 1, \ldots, m-1\}$ satisfies $|\{h \in H : h(x) = h(y)\}| = |H|/m$ for all $x \neq y \in \mathcal{U}$.

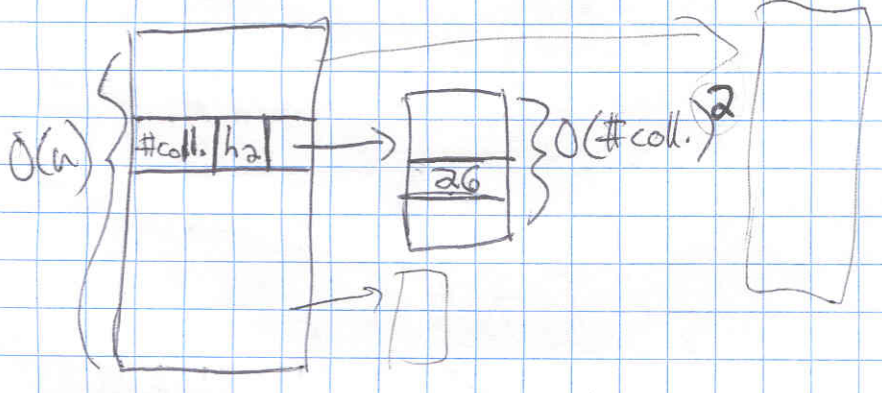
$\Rightarrow$ Choose $h \in H$ at random $\to$ get few collisions

E.g.: Write key k in base m: $k_0, k_1, \ldots, k_r$

Pick random vector in base m: $a_0, a_1, \ldots, a_r$

$$h_a(k) = \sum_{i=0}^{r} a_i k_i \bmod m.$$

Perfect hashing: 2-level universal hashing



Universal hash function h_1 determines slot in Table 1.

Pick second-level universal hash functions h_2 so that there are no collisions at 2nd level