Cost of unrefined levels of detail

- suppose level of detail $L-1$ has block size $\geq$ true B & level of detail L has block size $\leq$ true B
- monotonicity $\Rightarrow$ optimal cost for $B \geq$ cost @ level $L-1$.
- construction $\Rightarrow$ cost @ level $L \leq 4 \times$ cost @ level $L-1$.
- alignment may add another factor of 2

$\Rightarrow$ cost of layout at level $L \leq 8 \times$ optimal cost for B.

Cost of refined levels of detail (expected cost)

- define random variable

X_i = # blocks @ level i in random root-to-leaf path
= 1 + # edges straddling different blocks @ level i in random root-to-leaf path

- cost $X = (X_1 - 1) + (X_2 - 1) + \cdots + (X_L - 1) + 1$
$= X_1 + X_2 + \cdots + X_L - (L-1)$

- by linearity of expectation,

$E[X] = E[X_1] + E[X_2] + \cdots + E[X_L] - (L-1)$
$\leq E[X_1] + E[X_2] + \cdots + E[X_L]$.

- construction $\Rightarrow E[X_i] \geq 2 \times E[X_{i-1}] \Rightarrow$ geometric
$\Rightarrow E[X] \leq 2 \times E[X_L]$.

- unrefined analysis $\Rightarrow E[X_L] \leq 8 \times E[\text{optimal layout cost}]$
$\Rightarrow E[X] \leq 16 \times E[\text{optimal layout cost}]$.

Maximum cost: replace X_i with nonrandom variable $E[X_i]$ with X_i
no linearity of expectation
rest is the same

OPEN: dynamize? self-adjusting to guess distribution?

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