6.897
L17.2
Apr. 28, 2003

Analysis:

Space of K-funnel is $\geq K^2$ and $\leq cK^2$
(excluding output buffer of K^3)

- $S(K) = (1+\sqrt{K})\,S(\sqrt{K}) + K^2$. $(\Rightarrow \geq K^2)$
- $O(K)$ leaves, each costs $O(1)$
- root cost of K^2 dominates

Time: $O(\frac{K^3}{B} \log_M K + K)$

- consider level J at which size(J-funnel) $< M/4$ i.e. $cJ^2 < M/4$
  size(J^2-funnel) $\geq M/4$

(if you prefer, golden ratio $\frac{1+\sqrt{5}}{2}$)

$\Rightarrow$ can fit in cache one J-funnel $\rightarrow \boxed{\leq M/4}$
& one block per input buffer to J-funnel
$\hookrightarrow JB \leq \sqrt{M/4c}\sqrt{M} = M/\sqrt{4c}$ $\boxed{\leq M/2}$.

- size(J-funnel) $= \Theta(J^2) = \Omega(\sqrt{M}) = \Omega(B)$ (good)
- multiply by $J/B \Rightarrow J^3/B = \Omega(J)$.

- reading entire J-funnel + one block per input buffer (init.) costs $O(J^2/B) + O(J) = O(J^3/B)$

- when an input buffer empties, we recurse & lose all memory of this J-funnel
  but... the $\geq J^3$ elements filled in input buffer can pay for reload cost of $O(J^3/B)$.

- each element pays once for each buffer between J-funnels that it enters up path

$$\Rightarrow \frac{\lg K}{\lg J} + O(1) \leq \frac{\lg K}{\frac{1}{4}\lg M} = O(\log_M K).$$

- total cost therefore $O(\underbrace{\frac{K^3}{B}}_{\text{charged cost}} \underbrace{\log_M K}_{\#\text{ charges}} + \underbrace{K}_{\text{to get input lists started}})$

$$= O(\frac{K^3}{B} \log_{M/B} \underbrace{K/B}_{\Omega(\sqrt{K}) \text{ provided } K=\Omega(B^2)})$$

if $K = O(B^2) = O(M) \rightsquigarrow$ base case

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