Divide-and-Conquer

6.897
L16.1
Apr. 23, 2003

- major technique in cache-oblivious design
- algorithm divides problem recursively all the way to $O(1)$
- analysis can stop short & consider point when size $\le M$ or size $= O(B)$.

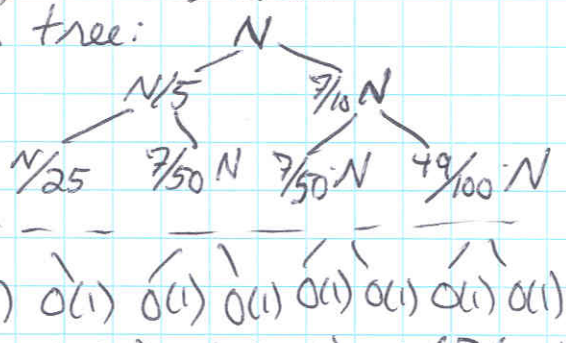
Median/Select (a simple example)

- linear-time worst-case selection: [Blum, Floyd, Pratt, Rivest, Tarjan 1973]
  1. conceptually partition array into $N/5$ 5-tuples — $\emptyset$
  2. compute median of each 5-tuple — parallel scans
  3. recursively compute median m of these medians — recursion
  4. partition array into $\le m$ and $> m$. — parallel scans
  5. count lengths and recurse into appropriate half — recursion
- recurrence for # memory transfers:

$$T(N) = T(N/5) + T(7/10\,N) + O(\lceil N/B \rceil)$$

↳ without this, we'd get time.

- what's the base case?
  - $T(O(1)) = O(1)$ ~as usual
    ↳ recursion tree:



← not all at same depth

↳ # leaves: $L(N) = L(N/5) + L(7/10 \cdot N)$

$$N^c = (N/5)^c + (7/10\,N)^c$$
$$1 = (1/5)^c + (7/10)^c$$
$$c \approx 0.8397803\ldots$$

↳ $T(N) \ge N^{0.83978\ldots} > N/B$ if $B \ge N^{0.16021\ldots}$ ☺

  - $T(O(B)) = O(1)$ ~stronger
    ↳ # leaves $= (N/B)^c = o(N/B)$.
    ↳ dominant cost is at root $\to O(\lceil N/B \rceil)$
    ↳ geometric by level
    ↳ total cost $= O(\lceil N/B \rceil)$. ☺

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