6.897
L14.3
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# Order queries in a list

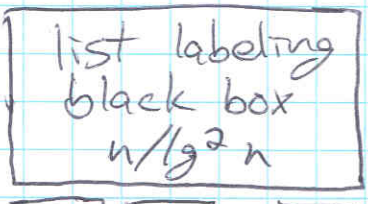
- maintain a linked list subject to
  - Insert element between two given nodes
  - Delete a given node
  - Order(x, y): which node comes first in the list?

## List labeling: slightly harder

- maintain tags for each node in linked list (monotonic) such that Order query can be answered by comparing tags

| Tag space | Time/update | |
|---|---|---|
| $\Theta(n)$ / $\Theta(n \lg n)$ | $O(\lg^2 n)$ | ← what we saw |
| $\Theta(n^2)$ or $\Theta(n^{1+\varepsilon})$ | $O(\lg n)$ | ← modification to thresholds |
| $\Theta(2^n)$ | $O(1)$ | ← trivial |

## Indirection can solve order queries using "implicit tags"




$\Theta$( $\lg^2 n$ | $\lg^2 n$ | ... | $\lg^2 n$ ) ← balanced search trees on $\Theta(\lg^2 n)$ elements

- label element with top list label ——— $\lg n$ bits
  + root-to-leaf path label (bottom) — $\lg\lg n$ bits
- use B-trees for bottom chunks
  ⇒ can afford to relabel all descendents whenever you touch a node
- bottom trees slow down updates to top

$O(1)$ amortized updates
worst-case possible [Dietz & Sleator - STOC 1987]
[Bender et al. - ESA 2002]

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