Ordered-File Maintenance

[Itai, Konheim, Rodeh 1981] [Bender, Demaine, Farach-Colton 2000]

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- given n elements in some order
- store them in that order in an array of size $O(n)$
- updates: – insert an element between 2 given elements
  – delete a given element

Naïve solution: no gaps — shift to make room
$\Theta(n)$ in worst case for Insert & Delete.

Better: $O(\lg^2 n)$ amortized per update (!)

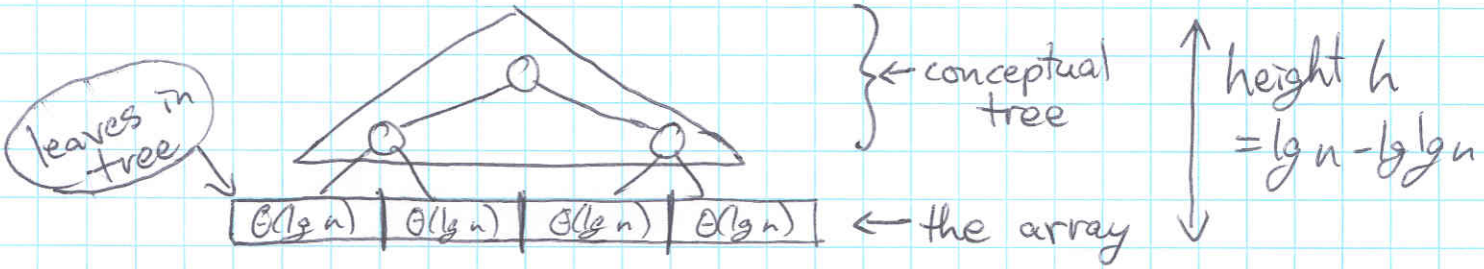
- can be made worst case [Willard 1992] [Bender, Cole, Demaine, Farach-Colton, Zito - ESA 2002]
- conjectured to be best possible (OPEN)

Rough idea:

- try to keep gaps evenly distributed
- upon update of an element, ensure not too crowded:
  – grow an interval around it...
  – until not too crowded on average
  – then evenly redistribute elements in interval
- the magic is in the amortization...

Tree view:

- build a (conceptual) complete binary tree on clusters of $\Theta(\lg n)$ slots



- each node in tree represents an interval of the array (of size $\Omega(\lg n)$)

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