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6.897
L1.2
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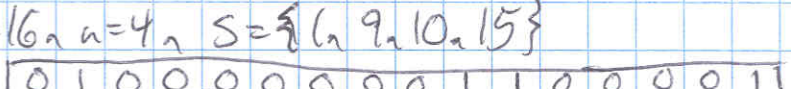
Where might $O(\lg \lg u)$ bound arise?
- binary search over $O(\lg u)$ things
- recurrence $T(u) = T(\sqrt{u}) + O(1)$

① Starting point: Bit vector

$\forall x \in \mathcal{U}$, $V_x = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$

e.g. $u = 16$, $n = 4$, $S = \{1, 9, 10, 15\}$



Insert/Delete — $O(1)$ time
Successor/Predecessor — $O(n)$ worst-case time

② Split universe into chunks

Carve universe of size u into $\sqrt{u}$ substructures of size $\sqrt{u}$.
$sub[0], sub[1], \ldots, sub[\sqrt{u}-1]$.

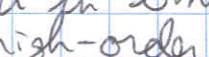
e.g.

$sub[i]$ represents $i\sqrt{u}, i\sqrt{u}+1, \ldots, (i+1)\sqrt{u}-1 \in \mathcal{U}$

Bit manip.

Write $x \in \mathcal{U}$ in binary using $\lg u$ bits
$high(x)$ = high-order half of bits = which substructure
$low(x)$ = low-order half of bits = index within substructure

e.g. $x = 9$



Insert(x) into substructure $high(x)$, mark as nonempty $O(1)$
Successor(x) — look within $sub[high(x)]$ } $O(\sqrt{u})$
— else find next nonempty substructure } $O(\sqrt{u})$
& return smallest element in it } $O(\sqrt{u})$

$O(\sqrt{u})$