6.897
L1.1
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Fixed-universe successor problem

- Maintain dynamic subset S of size n of the universe $\mathcal{U} = \{0, 1, \ldots, u-1\}$ of size u subject to
  - Insert ($x \in \mathcal{U} \setminus S$)
  - Delete ($x \in S$)
  - Successor ($x \in \mathcal{U}$): smallest elt. in S that's $> x$.
  - Predecessor ($x \in \mathcal{U}$)

- Balanced trees — $O(\lg n)$ /op. $\cdot$ $O(n)$ space in comparison model & pointer machine (arb. $\mathcal{U}$)
- → van Emde Boas (1975) — $O(\lg \lg u)$ /op. $\cdot$ $O(u)$ space on pointer machine [→ $O(n)$ on RAM]
- Mehlhorn, Näher, AH (1988) — $\Omega(\lg \lg u)$ /op on pointer machine
- → y-fast trees (Willard 1984) — $O(\lg \lg u)$ /op. $\cdot$ $O(n)$ space randomized on RAM.
- Beame & Fich (1999) — $\Omega\left(\min\left\{\frac{\log\log u}{\lg\lg\lg u}, \sqrt{\frac{\lg n}{\lg\lg n}}\right\}\right)$ in cell-probe model

  $O\left(\min\left\{\frac{\lg\lg u}{\lg\lg\lg u} \cdot \lg\lg u, \sqrt{\frac{\lg n}{\lg\lg n}}\right\}\right)$

- Andersson & Thorup (2000) — dynamize ↑ static on RAM, worst-case