Exam 1
Assigned: 03/15/2005
Due: 03/17/2005 (Midnight)

Requirements: *The exam is open-book and open-web. (Sources should be cited if you referred to websites or other books.) Take-home exams may not be discussed. No collaboration is allowed. You should include the following sentence in your exam write-up, which you need to sign physically or electronically:* `"I affirm that I did not discuss this exam or receive help from anyone else on it, and that I have disclosed the resources that I used in working on each problem. Signature: ___________ Date: ___________"`
Submission: *Submission deadline is 03/17/2005 Midnight. No extension will be granted. Submission can be an electronic version to 6869-submit@csail.mit.edu, or a hard-copy sent to 32D542.*

Problem 1 *Projection-1*

The following question uses the pinhole camera model and perspective projection given in Figure 1.4 and Equation (1.1) in Forsyth and Ponce.

(a) Show that under perspective, the projection of a 3D line is a 2D line in the image.

Hint: A 3D line can be written as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + \mu \begin{pmatrix} u_X \\ u_Y \\ u_Z \end{pmatrix}$$

where $\mu \in \mathcal{R}$, $u = (u_X, u_Y, u_Z)^T$ is a unit vector.

(b) Show that under perspective, the projected images (2D lines) of parallel lines in 3D intersect at a point. (i.e., vanishing point)

(c) Suppose the vanishing point for two parallel lines has the image coordinates (u_∞, v_∞). Show that the direction vector u for the 3D lines is given by

$$u = \frac{1}{\sqrt{u_\infty^2 + v_\infty^2 + f^2}} \begin{pmatrix} u_\infty \\ v_\infty \\ f \end{pmatrix}$$

where f is the focal length of the camera.

(d) Do the following models all preserve parallelness when projecting 3D parallel lines to 2D images? If yes, why? If not, point out which one(s) and why?

i) Affine ii) Weak perspective iii) Orthographic iv) Paraperspective

Problem 2 *Projection-2*

Solve exercise 2.9 in Forsythe and Ponce.

Problem 3 *Orthographic, weak, and paraperspective projection*

With the camera center at the origin, the image plane at $(z = -1)$ and a scene reference point at $(0, 1, 20)$ describe where scene points at $(2, 0, 20)$, $(3, 3, 20)$, and $(1, 0, 21)$ will project under (a) orthographic, (b) weak perspective, and (c) paraperspective projection.

See Section 2.3 in Forsythe and Ponce for 'scene reference point', weak and paraperspective projections.

Problem 4 *Image pyramids*

(a) If there are n pixels in an input image, how many pixels are there in a Laplacian pyramid? (Consider an infinite Laplacian pyramid which gives the upperbound of this number.)

What is this number for a steerable pyramid where each scale has 4 orientation bands?

(b) Unlike the QMF, the Laplacian and steerable pyramids offer overcomplete representations of an image. Why might it be desirable to have an overcomplete representation for early vision? Please explain in one or two sentences.

Problem 5 *Color - Metamerism*

Suppose you observe a monochromatic light of wavelength 500nm. What combination of monochromatic lights at wavelengths 450, 550, and 650 nm will be needed to give a perceptual match to the color of the 500 nm light? Suppose you wanted to match the 500 nm color, using lights of 410, 510, and 610nm, what amount of each of them is required? What is the significance of negative values of light? Use the color matching curves given in the homework. Suppose you had four lights, at 410, 510, 610, and 650 nm. What condition on the four lights will guarantee that the resulting color matches the 500nm light?

Problem 6 *Smoothing*

(a) A 1D Gaussian kernel for smoothing is $G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$. If an image is smoothed by $G_\sigma(x)$ twice, what is the single kernel that this smoothing operation is equivalent to?

(b) We can also use median filter to smooth noisy images. Is a median filter

i) Separable in x y directions?

ii) Implementable by a convolution?

iii) Isotropic?

iv) What is the main advantage of using median filter rather than using a Gaussian filter for smoothing?

Problem 7 *Bayesian noise removal*

Histograms of the outputs of bandpass filters applied to images typically have the form $p_x(x) = ce^{-\frac{|x|}{\gamma}}$, where $p_x(x)$ is the probability of observing bandpass filter output x, c is a normalization constant and we will assume γ=1.

Suppose the image is corrupted by additive Gaussian random noise so that we observe $y = x + n$, where n is a random variable with probability density $p_n(n) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{n^2}{2\sigma^2}}$. We seek an estimate, $\hat{x}$, of the uncorrupted bandpass filter coefficient.

Write the likelihood function, $p(y|x)$. Write the posterior probability, $p(x|y)$. Assume variance of additive Gaussian noise is 1. For each of the three observations y = 0.1, 1, and 10, calculate both the MAP estimate (the maximum of the posterior probability) and the MMSE estimate (the mean of the posterior probability) of x. You should use Matlab to calculate your answers to 2 decimal points of accuracy. Explain in words why these answers make sense.

Problem 8 *Bayesian theorem*

Solve problem 1.9 in Bishop chapter 1. (See hard-copy handout.)

Problem 9 *Minimizing risk*

Solve problem 1.11 in Bishop chapter 1. (See hard-copy handout.)

Problem 10 *Harris corner detector*

Consider the generalization of the Harris corner detector to 3-D images, i.e., to image data I that is a function of x, y, and z.

(a) For small displacements δx, δy and δz, find the matrix $\boldsymbol{A}$ such that the squared error image change as a function of displacement can be written as

$$(\delta x, \delta y, \delta z)\, \boldsymbol{A} \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}$$

(b) Is A positive semi-definite? Why or why not?

(c) What are the conditions on A for a good local feature?

(d) (Extra credit question)

The 2-D condition for cornerness is $R = \det(\boldsymbol{A})$ - 0.04 * $\text{trace}^2(\boldsymbol{A})$. Propose an analogous formula for 3-D corner-like points and explain why it should work.